THE MOTION OF A GAS AROUND A STRONGLY HEATED BODY

V. N. Zhigulev

UDC 533.601.18

Maxwell was one of the first to study the thermal slipping and radiometry effects. In particular, he suggested [1] that the thermal stresses which occur in a gas are important in an analysis of the radiometry effect. Interest in these problems has recently increased in connection with the problem of the slow motion of a strongly heated body in a gas. The paper by Galkin et al. [2], for example, is devoted to this question. However, the paper contains certain inaccuracies, and this means that the problem needs to be reconsidered. The present note* describes the classification and the general characteristics of the types of motion and gives a statistical example of the state of a nonuniformly heated gas.

1. A nonuniformly heated body in contact with a gas generates a macroscopic motion in the gas at a characteristic velocity u_0 (see, for example, [3]):

$$u_0 \sim \varepsilon \nu / L \sim \varepsilon c \,\mathrm{Kn} \tag{1.1}$$

where ν is the coefficient of kinematic viscosity, L is a characteristic length, c is the velocity of sound, $\Delta T/T \sim \varepsilon \leq 0$ (1), ΔT is a characteristic change in temperature; here and in what follows, we consider only cases where the Knudsen number $\mathrm{Kn} \sim l/L \ll 1$, where l is the mean free path. This phenomenon, which is usually called thermal slipping, is closely related to the radiometry effect.

If a uniformly heated body is immersed in a gas at a different temperature, the thermal Barnett stresses which are produced will, generally speaking, not be in equilibrium [2] and will give rise to a macroscopic movement of the gas at a velocity u_0' whose order of magnitude can be found by equating the sizes of the Barnett stresses. It will be seen below that the Reynolds number of these movements is of the order of unity or smaller. We thus have

$$u_0' \sim \varepsilon \nu / L \tag{1.2}$$

which agrees with the estimate of the thermal slipping velocity (1.1). This is a maximum estimate but in principle u_0' can in certain particular cases be zero or at least smaller than u_0 (see below).

2. From (1.1) and (1.2) we can conclude that a nonuniformly heated gas is in general characterized by a macroscopic movement with a velocity of the order of u_0; i.e., the magnitude of u_0 is a fundamental characteristic of the state of the gas. Thus the motion of a strongly heated body in a gas, when $\varepsilon \gg M_\infty^2$, must be classified according to the value of the parameter w, where

$$w = u_\infty / u_0 \tag{2.1}$$

3. We consider various cases which might occur.

(1) $w \gg 1$; the Reynolds number

$$Re = u_\infty L/\nu \gg \varepsilon \tag{3.1}$$

*The paper was presented on March 16, 1971 at the Thirteenth Siberian Seminar on Thermal Physics.

Institute of Theoretical and Applied Mechanics, Siberian Section, Academy of Sciences of the USSR, Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 95–98, July–August, 1972. Original article submitted April 5, 1971.

To the first approximation in the small parameter w^{-1}, we have motion of the gas in the usual Navier–Stokes situation with a slow Stokes movement. The effect of w^{-1} is like a small perturbation and the equation describing this should include the Barnett stresses since

$$\frac{\mu^2}{\rho T}\frac{\partial^2 T}{\partial x_i \partial x_j} \Big/ \mu \frac{\partial u_i}{\partial x_j} \sim \frac{1}{w} \tag{3.2}$$

(μ is the coefficient of viscosity).

(2) $w \sim 1$, $w \ll 1$. The characteristic velocity is of the order of u_0 (1.1); the condition $\varepsilon \gg M_\infty^2$ is automatically satisfied.

The Reynolds number

$$\mathrm{Re} = u_0 L/\nu \sim \varepsilon \tag{3.3}$$

The parameter $u_\infty L/\nu$ does not in this case have any relation to the Reynolds number and from the condition

$$u_\infty L/\nu \sim \varepsilon, \qquad u_\infty L/\nu \lll \varepsilon \tag{3.4}$$

The relative sizes of the Barnett stresses, Navier–Stokes stresses, and Eulerian terms in the momentum conservation equations are

$$\frac{\mu^2}{\rho T}\frac{\partial^2 T}{\partial x_i \partial x_j} \Big/ \mu \frac{\partial u_i}{\partial x_j} \sim 1 \tag{3.5}$$

$$\rho u_i u_j / \mu \frac{\partial u_i}{\partial x_j} \sim \varepsilon \tag{3.6}$$

The ratio of the Eulerian and thermal Navier–Stokes terms in the equation for the conservation of energy is

$$\rho R T \frac{\partial u_i}{\partial x_i} \Big/ \lambda \frac{\partial^2 T}{\partial x_i^2} \sim 1 \tag{3.7}$$

$$c_v \rho u \frac{\partial T}{\partial x_i} \Big/ \lambda \frac{\partial^2 T}{\partial x_i^2} \sim \varepsilon \tag{3.8}$$

where λ is the thermal conductivity.

Two subcases can be distinguished.

A. $\varepsilon \sim 1$. The flow Reynolds number is of the order of unity.

The equations for the conservation of momentum must include all the Eulerian, all the Navier–Stokes, and the temperature Barnett terms (all these terms are of the same order of magnitude $L^{-1}\rho c^2 \mathrm{Kn}^2$); the energy conservation equation must contain the Eulerian and Navier–Stokes terms (whose order of magnitude is $L^{-1}\rho c^3 \mathrm{Kn}$); the remaining terms in these equations obtained by the Chapman–Enskog method are of no significance.

B. $\varepsilon \ll 1$ (motion slower than Stokes type). Here

$$\frac{\Delta\rho}{\rho} \sim \frac{\Delta T}{T} \sim \varepsilon, \qquad \frac{\partial u_i}{\partial x_i} = 0, \qquad \frac{\partial^2 T}{\partial x_i^2} = 0 \tag{3.9}$$

and thus most of the Barnett terms with order of magnitude ε in the equations of motion can be cancelled. We have the usual situation with thermal slipping at low Reynolds numbers. If the temperature of the body is constant, then in general

$$u_0' \sim \varepsilon^n \nu / L \qquad (n > 1) \tag{3.10}$$

and the parameter w must be evaluated on the basis of (3.10). Thus ($w \sim, \ll \varepsilon$)

$$\mathrm{Re} \sim \varepsilon^n, \quad \partial^2 T / \partial x_i^2 = 0, \quad \partial u_i / \partial x_i = 0$$

and for the flow determination we have the Stokes situation with a known force (the nonlinear part of the thermal stresses is of the order of ε^n) on the right side of the momentum conservation equation.

This completes the classification and description of the general characteristics of the motions of strongly heated bodies in a gas.

4. From the nature of the estimate of u_0 ($u_0 \sim c$ Kn), we can conclude that when Kn ~ 1 the phenomena such as we have described acquire great importance. Hence, it is necessary to make a correct study of these effects for Kn $\ll 1$.

5. We consider the problem of the state of a gas situated between two parallel plates with very different temperatures T_1 and T_2 (i.e., $\varepsilon \sim 1$). The equations of motion of the gas then permit of the following exact solution.

In equilibrium the temperature of the gas varies as

$$\lambda \frac{dT}{dx} = \text{const}_1 \tag{5.1}$$

The momentum conservation equations can be integrated to give (see [1])

$$p + \frac{2\mu^2}{3\rho T}\left[\omega_3 \frac{d^2T}{dx^2} + \omega_5 \frac{1}{T}\left(\frac{dT}{dx}\right)^2\right] = \text{const}_2 \tag{5.2}$$
$$\left(\omega_3 = \alpha, \quad \omega_5 = \alpha \frac{d \ln \mu}{d \ln T} + \beta\right),$$

An interesting feature of this solution is that the pressure p in a nonuniformly heated gas is a variable.

An estimate of the total change in pressure between the plates 1 and 2 gives

$$\Delta p \sim \rho c^2 \, \text{Kn}^2 \tag{5.3}$$

i.e., when Kn $\sim 10^{-2}$ the relative effect is of the order of 10^{-4}.

It is possible that an experimental study of this fine effect will enable the dissipative coefficients λ and μ to be studied under static conditions.

In conclusion the author wishes to thank V. V. Struminskii for a useful discussion of the results.

LITERATURE CITED

1. S. Chapman and T. G. Cowling, The Mathematical Theory of Nonuniform Gases, Cambridge University Press, Cambridge (1952).
2. V. S. Galkin, M. N. Kogan, and O. G. Fridlender, "Some kinetic effects in flows in a continuous medium," Izv. Akad. Nauk SSSR, Mekhan. Zhidk. i Gaza, No. 3 (1970).
3. L. D. Landau and E. M. Lifshitz, The Mechanics of Fluids, Pergamon Press, New York (1959).